

Belarusian State University

Department of Energy Physics



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Tempus



## Temperature dependences of conductivity and Hall effect in semiconductors

The goal of this work is to study the method of determination of semiconductor parameters based on the measurement of the conductivity and Hall effect.

### Theoretical basics

*Statistics of electrons and holes in semiconductors.* Obtain an expression for the concentration of electrons in the conduction band. The number of electronic states in an elementary energy range (Fig. 1) can be represented as

$$dN = g(E) dE, \quad (1)$$

where the density of electronic states  $g(E)$  is equal to

$$g(E) = \frac{1}{2\pi^2} \left( \frac{2m_n^*}{\hbar^2} \right)^{\frac{3}{2}} (E - E_C)^{\frac{1}{2}}. \quad (2)$$

Here  $\hbar$  is reduced Planck's constant,  $E_C$  is the energy corresponding to the conduction band bottom, and  $m_n^*$  is the effective mass of density of states, which can be expressed through the components of the tensor of effective mass of charge carriers.

Similarly, for the density of states of holes near the top of the valence band:

$$g(E) = \frac{1}{2\pi^2} \left( \frac{2m_p^*}{\hbar^2} \right)^{\frac{3}{2}} (E_V - E)^{\frac{1}{2}}, \quad (3)$$

where  $E_V$  is the energy corresponding to the valence band top, and  $m_p^*$  is the effective mass of density of states in the valence band.

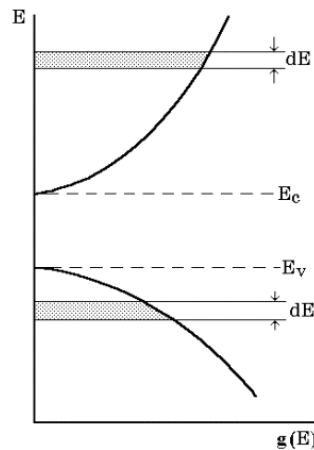


Fig. 1 – Density of electronic states in conduction and valence bands

It should be emphasized that the equations (2) and (3) are valid only for the states near the extrema of the electron dispersion law, i.e. at the bottom or the top of energy band. In the middle part of the band the exact form of the  $g(E)$  function is unknown.

Mathematical expectation of the number of electrons in the state with energy  $E$  is described by the Fermi-Dirac function:

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} \quad (4)$$

where  $E_F$  is the Fermi energy,  $k$  is the Boltzmann constant and  $T$  is the temperature.

Consequently, the number of electrons occupying the states in the energy range from  $E$  to  $E+dE$  in  $C$ -band:

$$dn = g(E) f(E) dE = \frac{1}{2\pi^2} \left(\frac{2m_n^*}{\hbar^2}\right)^{\frac{3}{2}} (E - E_C)^{\frac{1}{2}} \left(1 + \exp\left(\frac{E - E_F}{kT}\right)\right)^{-1} dE. \quad (5)$$

We shall measure an energy from the bottom of the conduction band. Then the total number of electrons in the conduction band is

$$\begin{aligned} n &= \frac{1}{2\pi^2} \left(\frac{2m_n^*}{\hbar^2}\right)^{\frac{3}{2}} \int_0^{E_C^{\max}} E^{\frac{1}{2}} \left(1 + \exp\left(\frac{E - E_F}{kT}\right)\right)^{-1} dE \cong \\ &\cong \frac{1}{2\pi^2} \left(\frac{2m_n^*}{\hbar^2}\right)^{\frac{3}{2}} \int_0^{\infty} E^{\frac{1}{2}} \left(1 + \exp\left(\frac{E - E_F}{kT}\right)\right)^{-1} dE = \frac{kT}{2\pi^2} \left(\frac{2m_n^*}{\hbar^2}\right)^{\frac{3}{2}} F_{1/2}\left(\frac{E_F}{kT}\right) \end{aligned} \quad (6)$$

Since the Fermi-Dirac function decreases very rapidly with increasing energy, we took the upper limit of integration equal to infinity. If the filling degree of the energy states in the conduction band by electrons is small ( $f(E) \ll 1$ ), we can neglect the unity in the denominator of the formula (6). Such is called semiconductor non-degenerate. For a non-degenerate semiconductor, formula (6) is simplified and leads to the following expression for the concentration of electrons in the conduction band:

$$n = \frac{1}{2\pi^2} \left(\frac{2m_n^* kT}{\hbar^2}\right)^{\frac{3}{2}} \exp\left(\frac{E_F}{kT}\right) = N_C \exp\left(\frac{E_F}{kT}\right), \quad (7)$$

where

$$N_C = \frac{1}{2\pi^2} \left(\frac{2m_n^* kT}{\hbar^2}\right)^{\frac{3}{2}} \quad (8)$$

is *effective density of states in the conduction band*. This term is associated with the fact that the total concentration of electrons distributed in the conduction band is the same as if  $N_C$  states with the energy corresponding to position of the conduction band bottom were occupied by electrons.

Similarly, one can calculate the concentration of holes in the valence band. Since the presence of a hole in the valence band is equivalent to the absence of an electron, the

mathematical expectation of the number of holes in a state with energy  $E$  is equal to  $1 - f(E)$ . Consequently, the number of holes in the energy range from  $E$  to  $E + dE$  in  $V$ -band:

$$dp = g_V(E) [1 - f(E)] dE. \quad (9)$$

Then:

$$\begin{aligned} p &= \frac{1}{2\pi^2} \left( \frac{2m_p^*}{\hbar^2} \right)^{\frac{3}{2}} \int_{E_V^{\min}}^{E_V} (E_V - E)^{\frac{1}{2}} \left[ 1 - \left( 1 + \exp\left( \frac{E - E_F}{kT} \right) \right)^{-1} \right] dE \cong [E_V = E_C - E_g = -E_g] \\ &\cong \frac{1}{2\pi^2} \left( \frac{2m_p^*}{\hbar^2} \right)^{\frac{3}{2}} \int_{-\infty}^{-E_g} (-E_g - E)^{\frac{1}{2}} \left( 1 + \exp\left( \frac{E - E_F}{kT} \right) \right)^{-1} dE \end{aligned} \quad (10)$$

If the filling degree of the energy states in the valence band by the holes is small, i.e.  $1 - f(E) \ll 1$ , (in other words, there is no degeneracy), the following expression for the concentration of holes is obtained:

$$p = \frac{1}{2\pi^2} \left( \frac{2m_p^* kT}{\hbar^2} \right)^{\frac{3}{2}} \exp\left( -\frac{E_g + E_F}{kT} \right) = N_V \exp\left( -\frac{E_g + E_F}{kT} \right), \quad (11)$$

where the *effective density of states in the valence band* is

$$N_V = \frac{1}{2\pi^2} \left( \frac{2m_p^* kT}{\hbar^2} \right)^{\frac{3}{2}}. \quad (12)$$

Multiply the equations (7) and (11):

$$np = N_C \exp\left( \frac{E_F}{kT} \right) N_V \exp\left( -\frac{E_F + E_g}{kT} \right) = N_C N_V \exp\left( -\frac{E_g}{kT} \right) \quad (13)$$

Equation (13) is called the *mass action law*. As can be seen from (13), knowledge of the carrier concentration of one sign (as example, electrons) allows to calculate the concentration of carriers of opposite sign (holes). It is obvious that (13) is valid only for non-degenerate semiconductors.

Shall use above relations to determine the concentration of charge carriers and the Fermi level position in an intrinsic semiconductor.

In intrinsic (undoped) semiconductor charge carriers can occur only due to the excitation of electrons from the valence band to the conduction band, so

$$n = p = n_i. \quad (14)$$

# Concentration of charge carriers in an intrinsic semiconductor is usually denoted  $n_i$  ( $i$  - intrinsic) ##.

Formula (14) is called the equation of electrical neutrality for the intrinsic semiconductor. Substituting (7) and (11) into (14), we obtain:

$$N_C \exp\left(\frac{E_F}{kT}\right) = N_V \exp\left(-\frac{E_g + E_F}{kT}\right). \quad (15)$$

Hence,

$$E_F = -\frac{E_g}{2} + \frac{kT}{2} \ln \frac{N_V}{N_C} = -\frac{E_g}{2} + \frac{3}{4} kT \ln \frac{m_p^*}{m_n^*}. \quad (16)$$

Thus, at  $T = 0$  the Fermi level for the intrinsic semiconductor is located in the middle of the band gap. The temperature dependence of the Fermi level in an intrinsic semiconductor is determined by the second term in equation (16). If the effective mass of holes in the valence band is more than effective mass of the electron in the conduction band (which happens often), the Fermi level shifts with increasing temperature to the bottom of the conduction band. In the opposite case the Fermi level shifts toward the valence band. The position of the Fermi level in intrinsic semiconductor depending on the temperature is shown schematically in Fig. 2.

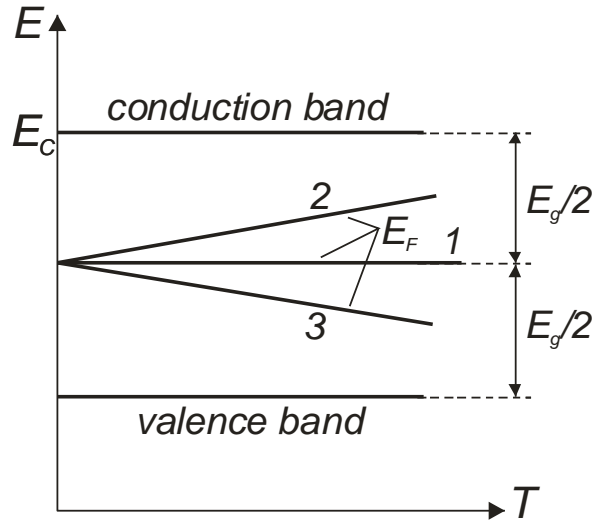


Fig. 2 – The position of the Fermi level in an intrinsic semiconductor:

$$1 - m_p^* = m_n^*; \quad 2 - m_p^* > m_n^*; \quad 3 - m_p^* < m_n^*.$$

For most semiconductors the hole effective mass is not much greater than the electron effective mass and the Fermi level shifts with temperature slightly. However, as example, for InSb  $m_p^*/m_n^* \approx 10$ , and the band gap is not large ( $E_g = 0.17$  eV). Therefore, in InSb at  $T > 450$  K the Fermi level enters the conduction band. At this temperature the semiconductor becomes degenerate.

*Fermi level and the charge carrier concentration in the doped semiconductor.* The position of the Fermi level in doped semiconductors can also be found from the condition of electrical neutrality of the crystal. For the n-type semiconductor, this condition has the form

$$n = N_d^+ + p, \quad (17)$$

where  $N_d^+$  is the concentration of ionized donors. Equation (17) indicates that the concentration of electrons in the conduction band is equal to the sum of concentrations of the holes in the valence band and the positively charged donors.

The concentration of ionized donors can be represented as follows:

$$N_d^+ = \frac{N_d}{2 \exp\left(\frac{E_F + E_d}{kT}\right) + 1}, \quad (18)$$

where  $E_d > 0$  is energy of the donor ionization,  $N_d$  is the total donor concentration. Equation (18) is reminiscent of the Fermi-Dirac function. The multiplier 2 before exponent is due to the fact that the electron localized at the donor can have two different spin directions. Therefore, the neutral state of the donor center has twice as large statistical weight compared with the ionized one.

Substituting (7), (11) and (18) in the electroneutrality equation, we obtain

$$N_C \exp\left(\frac{E_F}{kT}\right) = N_V \exp\left(-\frac{E_g + E_F}{kT}\right) + \frac{N_d}{2 \exp\left(\frac{E_F + E_d}{kT}\right) + 1}. \quad (19)$$

When the  $E_g$ ,  $N_C$ ,  $N_V$  values (which are unique characteristics of the semiconductor material and do not depend on its doping), as well as the dopant concentration  $N_d$  are known, one can solve (19) relative to  $E_F$  and substituting it into (7), (11) and (18), calculate the concentration of carriers in  $C$ - and  $V$ -bands and the concentration of ionized donors.

Consider the special case of low temperatures, when we can neglect the concentration of holes in the valence band (all the electrons in the conduction band are due to ionization of donors). In this case (19) takes the form

$$N_C \exp\left(\frac{E_F}{kT}\right) = \frac{N_d}{2 \exp\left(\frac{E_F + E_d}{kT}\right) + 1}. \quad (20)$$

Denote  $\exp\left(\frac{E_F}{kT}\right) = x$ , then

$$2N_C \exp\left(\frac{E_D}{kT}\right) x^2 + N_C x - N_d = 0. \quad (21)$$

Solving quadratic equation for  $x$ , we obtain

$$\exp\left(\frac{E_F}{kT}\right) = \frac{\exp\left(-\frac{E_D}{kT}\right)}{4} \left( \sqrt{1 + \frac{8N_d}{N_C} \exp\left(\frac{E_D}{kT}\right)} - 1 \right), \quad (22)$$

wherefrom

$$E_F = -E_D + kT \ln \left( \frac{1}{4} \sqrt{1 + \frac{8N_d}{N_C} \exp\left(\frac{E_D}{kT}\right)} - 1 \right). \quad (23)$$

In the particular case of low temperatures, when  $\frac{8N_d}{N_C} \exp\left(\frac{E_D}{kT}\right) \gg 1$ , we obtain

$$E_F = -\frac{E_D}{2} + \frac{1}{2}kT \ln\left(\frac{N_d}{2N_C}\right), \quad (24)$$

and electron concentration

$$n = \sqrt{\frac{N_C N_d}{2}} \exp\left(-\frac{E_D}{2kT}\right). \quad (25)$$

The relations (24) and (25) correspond to the temperature range, in which the ionization of donor impurity occurs.

If we now consider another special case of equation (23), namely  $\frac{8N_d}{N_C} \exp\left(\frac{E_D}{kT}\right) \ll 1$ , we obtain the following expressions for  $E_F$  and  $n$ :

$$E_F = kT \ln\left(\frac{N_d}{2N_C}\right), \quad (26)$$

$$n = N_d. \quad (27)$$

Equation (27) indicates that in this temperature range all impurities are already ionized, and the excitation of charge carriers from the valence band to the conduction band is still negligible.

Consider now the high-temperature region, when all donors are ionized. Electroneutrality equation in this case can be written as

$$n = p + N_d. \quad (28)$$

Writing the law of mass action in the form of

$$np = n_i^2, \quad (29)$$

we obtain a quadratic equation for  $n$ :

$$n^2 - N_d n - n_i^2 = 0, \quad (30)$$

the solution of which is

$$n = \frac{N_d}{2} \left( 1 + \sqrt{1 + \frac{4n_i^2}{N_d^2}} \right), \quad (31)$$

or

$$N_C \exp\left(\frac{E_F}{kT}\right) = \frac{N_d}{2} \left( 1 + \sqrt{1 + \frac{4n_i^2}{N_d^2}} \right), \quad (32)$$

wherefrom

$$E_F = kT \ln \left( \frac{N_d}{2N_C} \left( 1 + \sqrt{1 + \frac{4n_i^2}{N_D^2}} \right) \right). \quad (33)$$

Let us consider two limiting cases of equations (31) and (33). If  $4n_i^2 \ll N_D^2$ , we obtain

$$n = N_d, \quad (34)$$

$$E_F = kT \ln \left( \frac{N_d}{N_C} \right), \quad (35)$$

i.e. we come to the case of the donor impurity depletion. If  $4n_i^2 \gg N_D^2$ ,

$$n = n_i = \sqrt{N_C N_V} \exp \left( -\frac{E_g}{2kT} \right), \quad (36)$$

$$E_F = -\frac{E_g}{2} + \frac{3}{4} kT \ln \left( \frac{m_p^*}{m_n^*} \right), \quad (37)$$

i.e. we got the expressions derived above for the intrinsic semiconductor.

For a semiconductor containing an acceptor impurity, the temperature dependence of the Fermi level and carrier concentrations are calculated by the same way. Doing this, it is easy to see that in the temperature range corresponding to impurity ionization

$$E_F = -E_g + \frac{E_D}{2} - \frac{1}{2} kT \ln \left( \frac{N_A}{2N_V} \right), \quad (38)$$

and the hole concentration

$$p = \sqrt{\frac{N_V N_A}{2}} \exp \left( -\frac{E_A}{2kT} \right). \quad (39)$$

In the temperature range corresponding to the dopant depletion

$$E_F = -E_g - kT \ln \left( \frac{N_A}{N_V} \right), \quad (40)$$

$$p = N_A. \quad (41)$$

Changing of the Fermi level position with temperature in n-type (a) and p-type (b) semiconductors is shown in Figure 3.



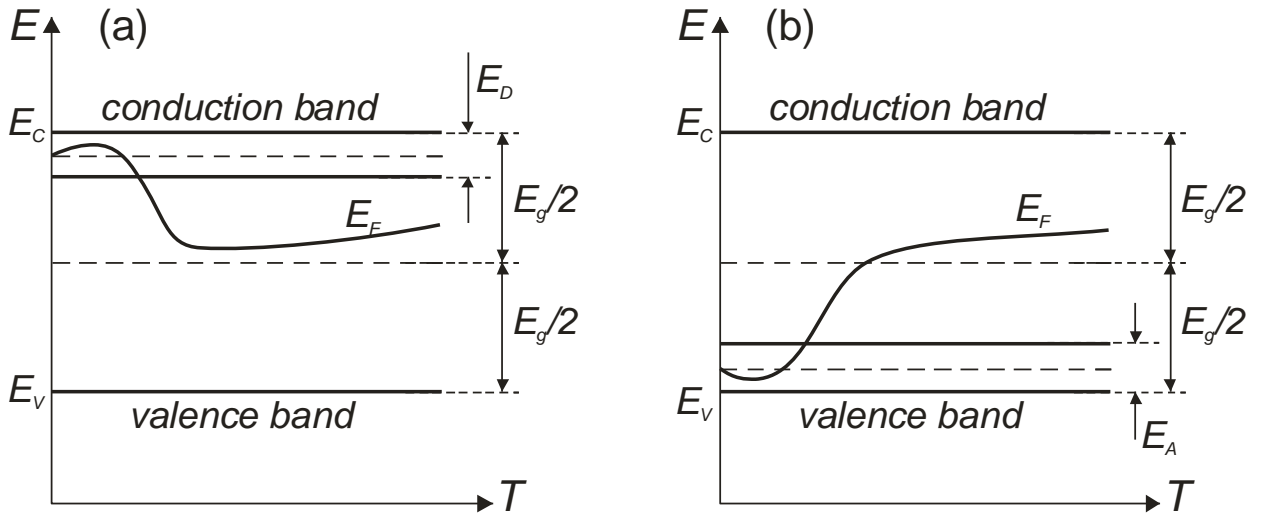


Fig. 3 – The temperature dependence of the Fermi level in n-type (a) and p-type (b) semiconductors

As follows from the above relations, the band gap of the semiconductor and the ionization energy of impurities can be determined from the temperature dependence of the charge carrier concentration. The most common method for determination of the charge carriers concentration is the measurement of the Hall effect. Since the electrical conductivity of the material (in the presence of one type of charge carriers) is given by

$$\sigma = en\mu_n \quad (\sigma = ep\mu_p), \quad (42)$$

the joint measurement of the Hall effect and conductivity makes it possible to determine both the concentration and mobility of the charge carriers.

*Hall effect.* Suppose that through the crystal having the shape of a rectangular plate the electric current flows under the influence of an electric field  $\mathbf{E}$ .

Let's place the sample into the uniform magnetic field  $\mathbf{B}$  perpendicular to the direction of current flow, as shown in Fig. 1. The Lorentz force equal to

$$\mathbf{F} = e\mathbf{v}\mathbf{B} \quad (43)$$

will act on a charge moving with the drift velocity  $\mathbf{v}$ . This force is directed perpendicular to the  $\mathbf{v}$  and  $\mathbf{B}$  vectors.

Under the action of this force the charge carriers will deflect to the side surfaces of the sample that will result in appearance of surface charges on these side surfaces. As a result of charge deviation by magnetic field, an electric field  $\mathbf{E}_H$  perpendicular to the  $\mathbf{E}$  и  $\mathbf{B}$  vectors will appear. *Occurrence of transverse electric field in the crystal with the flowing current under the influence of a magnetic field is called the Hall effect.* The Hall field will grow as long until the force acting by it on the charge carriers does not compensate the Lorentz force. The following expression will be true:

$$eE_H = e\mathbf{v}\mathbf{B}. \quad (44)$$

The emergence of the Hall field means the appearance between the lateral faces of the sample of the potential difference, which is called the Hall voltage  $V_H$ :

$$V_H = E_H b = \mathbf{v}\mathbf{B}b. \quad (45)$$

Inasmuch as  $\mathbf{j} = en\mathbf{v}$ ,

$$V_H = \frac{1}{en} jBb = RjBb. \quad (46)$$

The multiplier  $R$  in this equation is called the Hall coefficient and equal by definition

$$R = \frac{1}{en}. \quad (47)$$

When deriving these relations, we have not considered that the charge carriers are distributed by velocities. Account of this factor needs to write the expression (5) in the form

$$R = \frac{r}{en}, \quad (48)$$

where multiplier  $r$  is called Hall factor and depends on the mechanism of carrier scattering in the crystal. If the carrier scattering by acoustic phonons dominates,  $r = 3\pi/8$  (this usually holds at room temperature). If scattering by impurity ions is basic, then  $r = 315\pi/512 = 1.93$ .

From the above consideration it is seen that the measurement of the Hall effect enables one to determine the charge carriers in the crystal. If, in addition to measure the conductivity of the sample, it is possible to determine the carrier mobility.

Consider now the difference in Hall effect manifestation in n- and p-type semiconductors (Fig. 4).

In the cases of electron and hole semiconductors at the same orientation of external electric and magnetic fields, the directions of movement of charge carriers are opposed: holes move in the direction of the electric field, the electrons - against the electric field. Due to the opposite sign of the electron and hole charge the direction of the Lorentz force acting on them is the same and it rejects them in the same direction. Consequently, under identical experimental conditions, the direction of the Hall field and the sign of the Hall voltage are opposite for n- and p-type semiconductors. Thus, the measurement of the Hall effect allows to determine the conductivity type of semiconductor.

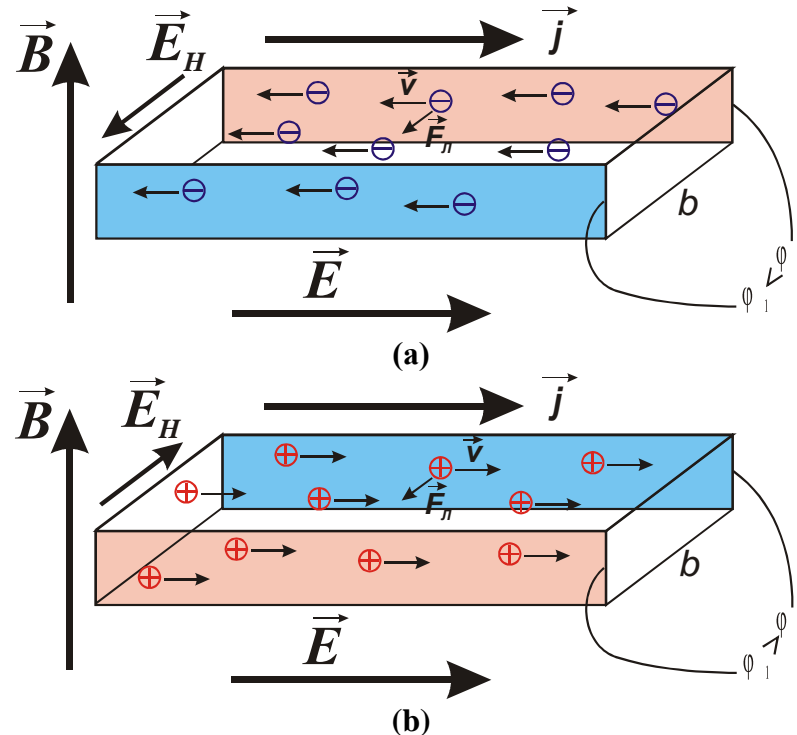


Fig. 4 – Schematic representation of the Hall effect in n-type (a) and p-type (b) semiconductors

Stop on the differences in the manifestation of the Hall effect in metals and semiconductors. They have a quantitative character and are determined by the following circumstance. In metals, where the carrier concentration is high (about  $10^{22} \text{ cm}^{-3}$ ), a large number of levels in the conduction band is filled with electrons. Since the thermal velocity of an electron in a crystal is given by

$$E - E_C = \frac{mv^2}{2}, \quad (49)$$

where  $E_C$  corresponds to the conduction band bottom, thermal velocity of carriers in metals are quite large and, therefore, free path time, mobility and drift velocity are small enough. In semiconductors, as a rule, only the closest to the band edge levels are filled, hence the thermal velocities of carriers in semiconductors are smaller than in metals. Therefore, free path time, mobility, and carrier drift velocity accept in semiconductors greater values than in metals. In accordance with (45), this means that *ceteris paribus* the Hall voltage takes larger values in semiconductors.

# Remark 1. For this reason, the magnetic field sensors based on the Hall effect are produced on the basis of semiconductor crystals with a high carrier mobility (n-GaAs, n-InSb). ##

*Methods of Hall voltage measurement.* The Hall voltage is often measured on plates of rectangular shape. In this case, as follows from the relations given above, to determine the Hall coefficient  $R_H$  it is necessary to measure the Hall voltage, the current flowing through the sample, the magnetic induction, and the geometric sizes of the sample.

In accordance with the theory of the Hall effect, the metallic contacts to the sample for measurement the Hall voltage must have a very small area to avoid distortion of the current flow in the sample. In practice, the metallic probes or alloyed contact of small size are used. Also side branches made by ultrasonic cutting or chemical etching are used (Fig. 4). One give a dumbbell shape to the sample and the side chains. In this case, technology of creation of contacts, metallization and alloying is simplified. Such contacts do not distort the current lines in the sample and due to the large area have a low contact resistance and low noise levels. The presence of several side branches allows to measure the resistivity of the sample simultaneously with the Hall voltage.

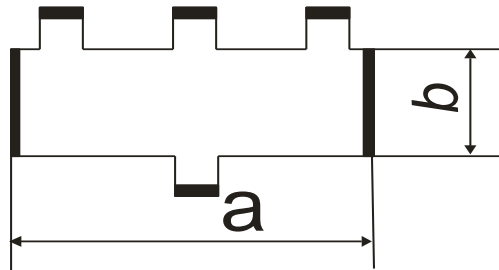


Fig. 4 – Example of the sample for the Hall effect measurement

The aspect ratio of the sample and contacts location can significantly affect the measured Hall voltage. Since the butt current contacts short Hall voltage, the sample should be long enough. In the case of symmetric arrangement of two Hall contacts one can use (27) to determine the Hall constant only when  $a/b \geq 4$ , otherwise it is necessary to introduce a correction function depending on the  $a/b$  ratio.

In plates of arbitrary geometrical shape the Hall voltage is measured similar to resistivity measurements by the van der Pauw method (Fig. 5). Electric current is passed through the contacts 1 and 3, and the potential difference is measured between the contacts 2 and 4 in the absence  $U_{24}(0)$  and presence  $U_{24}(B)$  of a magnetic field. Hall voltage is equal to

$$U_x = U_{BD} \left( \ominus \right) - U_{BD} \left( \ominus \right) = R_x \frac{I_{AC} B}{d}. \quad (50)$$

To determine the resistivity of the sample by the van der Pauw method, the measurements are carried out in the geometry shown in Fig. 6. The resistivity is determined by the formula

$$\exp\left(-\pi \frac{d}{\rho} R_{AB,CD}\right) + \exp\left(-\pi \frac{d}{\rho} R_{BC,AD}\right) = 1, \quad (51)$$

Where

$$R_{AB,CD} = \frac{|V_{CD}|}{I_{AB}}, \quad R_{BC,AD} = \frac{|V_{AD}|}{I_{BC}}. \quad (52)$$

It is possible to determine the Hall voltage, and hence the Hall coefficient by at least four different ways, using DC or AC current, and constant or alternating magnetic field.

Hall voltage measurement using a constant magnetic field and direct current is the easiest and most common. From a DC power source through the rectangular sample a direct current is passed. The sample is placed between the poles of the magnet. Change in the magnetic induction in the working gap of the magnet can be done by changing the distance between the poles of a magnet, varying electromagnet current and applying magnetic concentrators.

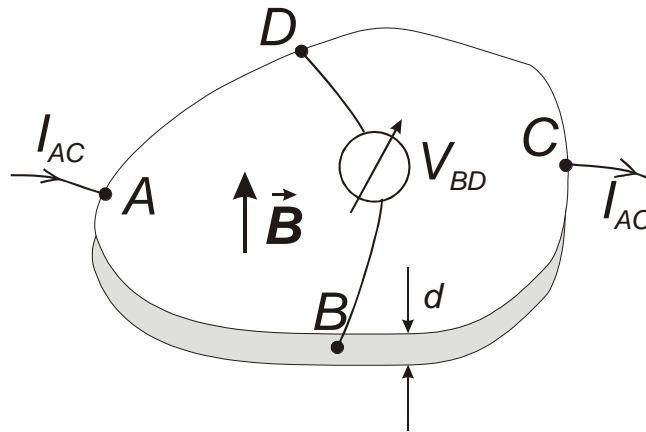


Fig. 5 – Scheme for the Hall effect measuring in a planar sample by the van der Pauw method

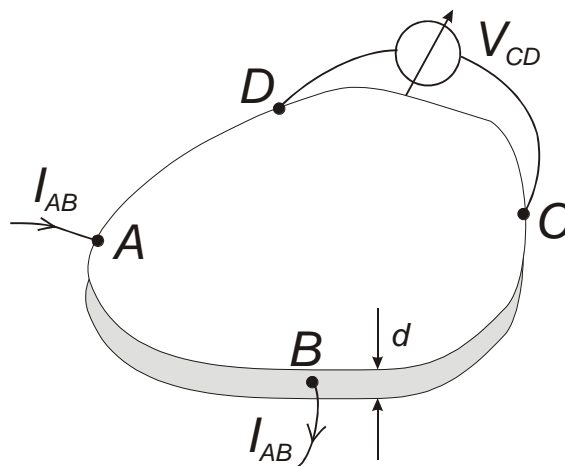


Fig. 6 – Scheme for the resistivity determination in a planar sample by the van der Pauw method

*Phenomena accompanying the Hall effect measurements.* For correct measurement of the Hall effect, it is necessary to consider a number of related physical phenomena that can contribute to systematic and random errors in the measurements. These phenomena include: cross thermogalvanomagnetic, thermogalvanic and thermomagnetic effects.

*Transverse thermogalvanomagnetic effect* occurs in a sample with a current placed in a magnetic field and consists in the appearance of a temperature gradient in direction perpendicular to the magnetic field and current. If, as in the case of the Hall effect, magnetic induction vector is directed along the  $z$  axis, and the electric current – along the  $x$  axis, a temperature gradient proportional to the magnetic induction and electric current density arises along the  $y$  axis:

$$\frac{\partial T}{\partial y} = -k_{tgm} B_z j_x, \quad (53)$$

where  $k_{tgm}$  is proportionality factor.

Physically, transverse thermogalvanomagnetic effect is due to the fact that the magnetic field acts on the "hot" (fast) charge carriers with greater force than on the "cold" (slow) ones. Therefore, the spatial separation of the charge carriers by velocities occurs. As a result of the energy exchange between the carriers and the lattice a temperature gradient occurs. The sign of this gradient is independent of the type of charge carriers, and depends on the direction of magnetic field and current just as the polarity of the Hall voltage. Therefore, a thermal emf occurring due to the temperature gradient, is summed with the Hall voltage and can not be separated from it when the direction of the current or magnetic field are changed

*Thermogalvanic* effect occurs under the following conditions. If the sample in which a heat flow passes in the  $x$  direction is placed in a magnetic field oriented along the  $z$  axis, the electric field occurs in the  $y$  direction. In the presence of a temperature gradient a preferred motion of "fast" charge carriers from the "hot" end of the sample to the "cold" one takes place.

The arising electric field is proportional to the temperature gradient and the magnetic induction:

$$E_y = -k_{tg} B_z \frac{\partial T}{\partial x}, \quad (54)$$

where  $k_{tg}$  is proportionality factor.

Under these conditions, the temperature gradient  $\partial T/\partial y$  proportional to  $B_z$  and  $\partial T/\partial x$  arises:

$$\frac{\partial T}{\partial y} = \pm k_{tm} B_z \frac{\partial T}{\partial x}, \quad (55)$$

where  $k_{tm}$  is proportionality factor.

This effect is called *thermomagnetic*. Sign of the temperature gradient depends on the type of charge carriers.

Thus, thermal electromotive forces arising due to thermogalvanomagnetic, thermogalvanic, and thermomagnetic effects occur together with the Hall voltage and are added to it. The temperature gradient  $\partial T/\partial x$  usually exists on the sample during the Hall measurement due to current flow through the contacts to the sample and release at them a certain amount of heat. Sign of the temperature gradient  $\partial T/\partial x$  is determined by the Peltier effect and non-ideality of current contacts and, hence, depends on the direction of current flow through the sample. Consequently, the signs of thermogalvanic and thermomagnetic effects depend on the direction of current flowing through the sample.

Furthermore, the existence of a temperature gradient  $\partial T/\partial x$  may be caused by the non-uniform heating of the sample (e.g., during the temperature measurement) or by its heterogeneity.

Estimation of the these effects contributions shows that the most important is the transverse thermogalvanomagnetic effect for which the thermopower can reach 10 % of the Hall voltage. Other effects less significant and play a more important role in metals.

Thus, when measuring the Hall voltage, the measured potential difference at the contacts consists of four components: the Hall voltage  $U_H$ , thermopower of transverse thermogalvanomagnetic effect  $U_{tgm}$ , thermopower of thermogalvanic effect  $U_{tg}$  and thermopower of thermomagnetic effect  $U_{tm}$ . There is another component which also affects the measured potential difference. This is a voltage  $U_0$  arising due to inaccurate location Hall contacts against each other, i.e. the at different sample equipotential surfaces.

In general, the potential difference between the contacts is equal to the sum of the five components. To get rid of parasitic EMF in the measurement of the Hall effect, the directions of the magnetic induction and the current are changed to the opposite. Assuming that the longitudinal temperature gradient  $\partial T/\partial x$  is due to non-uniform heating of the sample by an external heater and consequently does not depend on the direction of current flow through the sample, it is possible to write the system of equations for the potential difference which corresponds to all possible combinations of directions of current and magnetic fields:

$$\begin{array}{ll} \text{for } +I, +B & U_1 = U_H + U_{tgm} + U_{tg} + U_{tm} + U_0; \\ \text{for } -I, +B & U_2 = -U_H - U_{tgm} + U_{tg} + U_{tm} - U_0; \\ \text{for } -I, -B & U_3 = U_H + U_{tgm} - U_{tg} - U_{tm} - U_0; \\ \text{for } +I, -B & U_4 = -U_H - U_{tgm} - U_{tg} - U_{tm} + U_0. \end{array}$$

After simple transformations, we obtain

$$U_H + U_{tgm} = (U_1 - U_2 + U_3 - U_4)/4. \quad (56)$$

Thus, by performing measurements at four different directions of the magnetic field and the current through the sample, one can be exclude from the of the measurements all parasitic voltages except the thermopower of transverse thermogalvanomagnetic effect

It is not always possible to eliminate the influence of non-equipotentiality of contacts, especially for high-resistance samples. When sufficiently large currents are passing through the high sample, the voltage drop along the sample can be quite large. If this potential difference is much greater than the Hall voltage, the measurement accuracy may fall so that will make measurement impossible at all. Therefore, one seek not only to reduce a non-equipotentiality of contacts due to their precise setting against each other, but also realize their electrical balance.

### **Description of the laboratory setup**

For the measurement of the Hall effect and resistance of the sample the closed-cycle cryogen-free cryostat system CFM (Cryogenic Ltd., London) is used. This setup allows to do the measurements in the temperature range 1.7 – 305 K, stabilizing the temperature with accuracy of 0.005 K. The maximal magnetic field equals to 8 T. Measurable DC voltages are from 1  $\mu$ V to 211 V, DC currents – from 10 aA to 105.5 mA.

### **Tasks and the order of work execution**

1. Install the sample in the measuring cell and connect to the measuring circuit.
2. To measure the electrical conductivity and the Hall effect at room temperature.
3. Repeat the measurements at different temperatures.

4. To extract the maximum possible information from the obtained array of data.

### **Test questions**

1. Get the relation (2) for the density of electronic states for the case of an isotropic crystal.
2. Calculate the temperature at which exactly half of the phosphorus atoms are ionized in a silicon crystal ( $E_D = 45$  meV).
3. Explain why the vast majority of semiconductor materials the effective mass of holes is greater than the effective mass of electrons.
4. The concentration of phosphorus atoms in the first silicon wafer is twice higher than the second one. What is the relationship between the hole concentrations in these wafers?
5. Why magnetic field sensors based on the Hall effect is not made on the basis of metals?
6. Can conductivity of semiconductor decrease with increasing temperature?

### **Instructions for report execution**

The report should contain:

1. Title of the work.
2. Name of students who performed the work.
3. Introduction, where the aim of investigation is formulated.
4. Short summary of the theory explaining the physical nature of the investigated effects, meaning of physical quantities and measured dependencies.
5. Methods of investigation: description of the experimental setup and the objects under investigation, as well as estimation of inaccuracies should be given.
6. Experimental results and their discussion. Based on the experimental data, the main regularities should be marked and physical phenomena underlying them should be analyzed.
7. Conclusions. Critical analysis of the results should be contained here.

### **Recommended literature**

1. Шалимова, К.В. физика полупроводников / К.В. Шалимова. – М.: Энергоатомиздат, 1985. – 392 с.
2. Поклонский, Н.А. Статистическая физика полупроводников / Н.А. Поклонский, С.А. Вырко, С.Л. Поденок. – М.: КомКнига, 2005. – 264 с.